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# The hoop conjecture and cosmic censorship in the brane-world

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## Abstract

The initial data of gravity for a cylindrical matter distribution confined on the brane is studied in the framework of the single brane Randall–Sundrum scenario. We numerically found that the sufficiently thin configuration of matter leads to the formation of the marginal surface on the brane in the Randall–Sundrum model, even if the configuration is infinitely long. This means that the hoop conjecture proposed by Thorne does not hold in the Randall–Sundrum scenario; Even if a mass  $M$  does not get compacted into a region whose circumference ( $C$ ) in every direction is  $C \lesssim 4\pi GM$ , black holes with horizons can form on the brane-world of the Randall–Sundrum scenario.

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Assuming 4-dimensional general relativity and physically reasonable conditions on the matter fields, Thorne has proven that there is no marginal surface in the system of a cylindrical distribution of matter fields [1]. In his proof, the marginal surface means a cylindrically symmetric spacelike 2-surface such that the expansion of the outgoing null normal to this surface vanishes. This result, together with the Newtonian analogy, has led to the so-called hoop conjecture for the necessary and sufficient condition on

black hole formation; *Black holes with horizons form when and only when a mass  $M$  gets compacted into a region whose circumference in every direction is  $C \lesssim 4\pi GM$  [1].* The converse of the hoop conjecture gives a criterion of naked singularity formation; *If a mass  $M$  forms a singularity but does not get compacted into a region whose circumference in every direction is  $C \lesssim 4\pi GM$ , then the singularity will be naked.* Then the hoop conjecture is closely related to the cosmic censorship proposed by Penrose [2] which states, roughly speaking, that the gravitational collapse in a physically reasonable situation does not lead to naked singularities. If the hoop conjecture is really true in our universe, and if a singularity formed by physically reasonable gravitational collapse is not confined

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within a region of  $C \lesssim 4\pi GM$ , it is a counter example for the cosmic censorship.

No counter example for the hoop conjecture has been presented. Indeed, some numerical works have been done to confirm the hoop conjecture. The numerical simulations by Nakamura et al. strongly suggest that a highly elongated axisymmetric cold fluid forms a spindle naked singularity [3]. Later, Shapiro and Teukolsky also showed that the same is true for collisionless particle systems [4]. These numerical works suggest that the hoop conjecture will be true in 4-dimensional general relativity.

However, strictly speaking, we do not know whether general relativity can describe strong gravity in our real universe even for classical situations. We have no experimental evidence for it. If general relativity is inapplicable to the situation of the strong gravity, it again becomes a non-trivial issue whether a highly elongated spindle gravitational collapse could form naked singularities or not.

As an alternative theory of gravity, Randall and Sundrum (RS) recently proposed a scenario of the compactification of a higher dimension without compact manifold [5]. They considered 5-dimensional spacetimes with negative cosmological constant  $\Lambda < 0$  including a single 3-brane with the positive tension  $\lambda$ . In their scenario, all the physical fields, except for gravity, are assumed to be confined on the RS brane and the gravity is governed by 5-dimensional Einstein gravity. Using the fine tuning  $\sqrt{-6/\Lambda} = 1/\lambda =: l$ , they showed that even without a gap in the Kaluza–Klein spectrum, 4-dimensional Newtonian and general relativistic gravity on the brane is reproduced to more than adequate precision [5,6]. The deviation in the gravitational force from the Newtonian one appears in the short scale less than  $l$  [5,7]. Since experimental tests have already proved the  $1/r^2$  corrections to the Newtonian gravitational potential up to the sub-millimeter order [8], the length scale of  $l$  must be less than millimeter scale.

In the RS scenario, the cancellation of the long range forces due to the negative cosmological constant and the brane tension reproduces the 4-dimensional gravity on the RS brane. However, the sufficiently short range gravity (about less than the scale  $l$ ) seems to be so insensitive to the cosmological constant that the 5-dimensional nature of the gravity may appear and become important for the formation of black

holes on the RS brane. In fact within the limits of linear analysis, it has been proven that the gravity in RS model at short distance is 5-dimensional (see, e.g., [9]). If the results of the linear analysis can apply to the problem of the formation of black holes on the RS brane, we may imagine the possibility of thin spindle-like black holes and even infinitely long ones on the RS brane, considering the existence of black string solutions in 5-dimensional Einstein gravity [10]. Then we are lead to the guess that the hoop conjecture on the RS brane may not be valid in the scale less than  $l$ .

Of course this discussion should be justified by the fully non-linear analyses, since the formation of a black hole is a fully non-linear phenomenon. Moreover in the case of the cylindrical matter distribution on the RS mode, two different types of singular sources (a singular 3-brane and a singular matter distribution on it) interact with each other in complex way. Then the general relativistic analysis without any weak field assumption is necessary to confirm the validity of the hoop conjecture on the RS brane.

In order to get an insight into the black hole formation and the hoop conjecture in the RS scenario, we consider a cylindrically symmetric matter distribution on the brane and formation of marginal surfaces. In this Letter, we concentrate a time-symmetric initial data which is a 4-dimensional spacelike hypersurface embedded in the whole spacetime with vanishing extrinsic curvature [11] as a first step. The line element of the hypersurface we adopt here is

$$dl^2 = \phi^2(R, \xi) [dR^2 + R^2 d\varphi^2 + dz^2 + e^{2\xi/l} d\xi^2], \quad (1)$$

where the coordinate  $\xi$  corresponds to the extra dimension.

The conformal factor  $\phi$  which is determined by the initial value constraints of 5-dimensional Einstein gravity, i.e., the Hamiltonian constraint and momentum constraints. Since we concentrate on the time symmetric initial data, the momentum constraints are satisfied trivially. On the other hand, the Hamiltonian constraint is given by

$$\left\{ \partial_R^2 + \frac{1}{R} \partial_R + e^{-2\xi/l} \left( \partial_\xi^2 - \frac{1}{l} \partial_\xi \right) \right\} \Omega - \frac{2}{l^2} \Omega (\Omega^2 + 3e^{-\xi/l} \Omega + 3e^{-2\xi/l}) = 0, \quad (2)$$

where we have introduced a new variable defined by  $\Omega := \phi - e^{-\xi/l}$ .

The existence of the brane and the matter fields is taken into account by the boundary condition at  $\xi = 0$ . By the Israel's prescription [12], we can easily see that the extrinsic curvature of the brane in the initial hypersurface has discontinuity which is related to the tension of the brane and the stress-energy tensor of the matter fields. Following the RS model, we impose  $Z_2$ -symmetry with respect to  $\xi = 0$  [13].<sup>1</sup> Then the boundary condition on the brane  $\xi = 0$  is given by

$$\partial_\xi (e^{2\xi/l} \Omega) + \frac{\Omega^2}{l} + \frac{4}{3} \pi G_5 \rho(R) (\Omega + 1)^2 = 0, \quad (3)$$

where  $G_5 = Gl$  is the 5-dimensional Newton's gravitational constant [14],  $\rho(R)$  is the energy density of the matter fields. The 4-dimensional Minkowski spacetime ( $\Omega = 0$ ) is realized on the brane when  $\rho(R) = 0$ .

Because of the ambiguity in the original statement by Thorne, there are many proposals and attempts to prove the hoop conjecture [15]. In this Letter, we concentrate on the situation in which the support of the energy density  $\rho(R)$  is in the infinite cylinder with the coordinate radius  $R_s$ . Further, as a definition of the mass, we adopt the total proper mass

$$M := 2\pi \int_0^{R_s} dR R \int_{-L}^L dz \phi^3(R, 0) \rho(R) \quad (4)$$

within the cylinder of the coordinate radius  $R_s$  and a finite coordinate length  $2L$ . As the definition of the circumference, we adopt the proper length

$$C := 2 \int_{-L}^L dz \phi(R_s, 0) + 4 \int_0^{R_s} dR \phi(R, 0) \quad (5)$$

of this cylinder.

We are interested in the formation of a marginal surface in a situation of  $C > 4\pi GM$ , and can easily check that the inequality  $C > 4\pi GM$  holds for arbitrary  $L$  if and only if the following inequality is satisfied:

fied:

$$1 \geq \frac{4\pi^2 G_5}{l \phi(R_s, 0)} \int_0^{R_s} dR R \phi^3(R, 0) \rho(R). \quad (6)$$

It should be noted that this inequality gives an upper bound on the line energy density of the cylindrical matter field.

If the existence of future null infinity similar to the AdS or Minkowski spacetime and further the global hyperbolicity in the causal past of the future null infinity are guaranteed, the formation of a marginal surface in the initial data might mean the formation of a black hole with horizon. Then we may regard that the initial data, in which there is a marginal surface and the inequality (6) holds, as a counter example of the hoop conjecture.

There are two kinds of the marginal surfaces on the initial data: one is defined by the “null” rays confined in the brane and the other is defined by the null rays which propagate in the whole spacetime including the bulk. We call these two marginal surfaces brane-MS and bulk-MS, respectively. As commented in Ref. [11], these two marginal surfaces have different physical meanings from each other. Brane-MS is the marginal surface for all the physical fields confined on the brane. On the other hand, bulk-MS is for all the physical fields including gravitons which propagate in the whole 5-dimensional spacetime. Brane-MS is not concerned with the causal structure of the whole spacetime but bulk-MS is.

Brane-MS for the cylindrical matter distribution is a cylindrical 2-surface specified by  $R = \text{const}$  on the brane where the expansion of the null rays normal to the surface vanishes. In this case, the null rays are defined by the induced metric on the brane. On the time symmetric initial hypersurface, the condition of vanishing expansion of these null rays is equivalent to  $\partial_R (R \phi^2)|_{\xi=0} = 0$ . This is the equation for the coordinate radius of the brane-MS.

On the other hand, bulk-MS for the cylindrical matter distribution is a cylindrical spacelike 3-surface defined in the whole of a 4-dimensional spacelike hypersurface, on which the expansion of the outgoing null rays normal to the 3-surface vanishes. In this case, the null rays defined by the metric on the whole spacetime. On the time symmetric initial hypersurface, bulk-MS is expressed by a curve in  $(R, \xi)$ -plane, which inter-

<sup>1</sup>  $Z_2$ -symmetry is often imposed in higher-dimensional models in particle physics to avoid a problem of the chirality of the matter fields on the brane.

sects the axes  $R = 0$  and  $\xi = 0$ . To find this curve, we introduce spherical polar coordinate variables  $r := \sqrt{R^2 + l^2(e^{\xi/l} - 1)^2}$  and  $\theta := \tan^{-1}\{R/l(e^{\xi/l} - 1)\}$ . Then a cylindrical spacelike 3-surface will be specified by the function  $r = r(\theta)$ . The condition of the vanishing expansion leads to an ordinary differential equation for  $r(\theta)$  as

$$\begin{aligned} \frac{d^2 r}{d\theta^2} = & -\frac{3}{r^2} \left( \frac{dr}{d\theta} \right)^3 \partial_\theta \ln \phi + \left( \frac{dr}{d\theta} \right)^2 \left( \frac{2}{r} + 3\partial_r \ln \phi \right) \\ & - 3 \frac{dr}{d\theta} \partial_\theta \ln \phi + r + 3r^2 \partial_r \ln \phi \\ & + r \left\{ 1 + \frac{1}{r^2} \left( \frac{dr}{d\theta} \right)^2 \right\} \left( 1 - \frac{1}{r} \cot \theta \frac{dr}{d\theta} \right). \end{aligned} \quad (7)$$

The boundary conditions both at  $\theta = 0$  and at  $\pi/2$  to this equation are given by  $dr/d\theta = 0$ . We search for solutions of Eq. (7) numerically by the shooting method.

We solve Eq. (2) numerically by the finite difference method. The numerically covered region is  $R_{\max} \geq R \geq 0$  and  $\xi_{\max} \geq \xi \geq 0$ . Then we need to specify the boundary conditions at four kinds of numerical boundaries;  $R = 0$ ,  $R = R_{\max}$ ,  $\xi = 0$  and  $\xi = \xi_{\max}$ . We impose  $\partial_R \Omega = 0$  at  $R = 0$  and Eq. (3) at  $\xi = 0$ . To fix the boundary conditions at  $R = R_{\max}$  and at  $\xi = \xi_{\max}$ , we assume that the system is isolated, i.e., the space approaches to that of the AdS spacetime for  $r \rightarrow \infty$ . Therefore, the solution should behave as those to linearized Eq. (2) near the numerical boundaries  $R = R_{\max}$  and  $\xi = \xi_{\max}$ .

The linear solution to Eq. (2) is obtained as follows: regarding  $|\Omega| = O(G_5 l \rho) \ll 1$ , we derive the linearized version of Eqs. (2) and (3) and solve this linearized equation with a singular line source  $\rho(R) = \sigma_L \delta(R)/2\pi R$ , where  $\sigma_L$  is the line energy density of the line source. The linear solution  $\Omega = \Omega_L$  is then given by

$$\begin{aligned} \Omega_L(R, \xi) := & -\frac{2G_5 \sigma_L}{l} \left\{ e^{-2\xi/l} \ln \left( \frac{R}{R_c} \right) \right. \\ & \left. - \frac{l}{3} \int_0^\infty dm u_m(l) u_m(l e^{\xi/l}) K_0(mR) \right\}, \end{aligned} \quad (8)$$

where  $R_c$  is an integration constant which corresponds to the freedom to rescale  $\xi$ ,  $K_0(x)$  is the modified Bessel function of the 0th kind, and  $u_m(\psi)$  is a

combination of the spherical Bessel functions of the first and second kinds;

$$\begin{aligned} u_m(\psi) = & \sqrt{\frac{2(ml)^4}{\pi((ml)^2 + 1)}} \\ & \times m\psi (n_1(ml) j_2(m\psi) - j_1(ml) n_2(m\psi)). \end{aligned} \quad (9)$$

Using the linear solution (8), we impose the following boundary conditions at these numerical boundaries:

$$\partial_R \left\{ \frac{\Omega(R, \xi)}{\Omega_L(R, \xi)} \right\} = 0, \quad \text{at } R = R_{\max}, \quad (10)$$

$$\partial_\xi \left\{ \frac{\Omega(R, \xi)}{\Omega_L(R, \xi)} \right\} = 0, \quad \text{at } \xi = \xi_{\max}. \quad (11)$$

These boundary conditions guarantee that the numerical solution  $\Omega$  behaves as the linear solution  $\Omega_L$  near the numerical boundaries  $R = R_{\max}$  and  $\xi = \xi_{\max}$ .

To guarantee that our cylindrical matter distribution is an isolated system, further,  $|\Omega|$  should be much smaller than unity in the vicinity of these numerical boundaries. However the boundary conditions (10) and (11) impose no constraint on the amplitude of  $\Omega$  itself at these boundaries and hence these boundary conditions are not sufficient. This is accomplished by choosing the line energy density of the system considered here to be sufficiently small. As will be shown below, this is naturally realized in a situation in which the inequality (6), or equivalently,  $C > 4\pi GM$  holds.

To derive the numerical solutions, we consider the following energy density  $\rho(R)$ ; for  $R < R_s$

$$\rho(R)(\Omega + 1)^2|_{\xi=0} = \frac{3\sigma}{\pi R_s^2} \left\{ \left( \frac{R}{R_s} \right)^2 - 1 \right\}^2, \quad (12)$$

while  $\rho(R) = 0$  elsewhere, where  $\sigma$  is a constant which corresponds to the line energy density of this system. We choose  $l = 1$  (this is regarded as a choice of the unit) and then set the numerical boundaries to be  $R_{\max} = \xi_{\max} = 2$ . The number of numerical grids for Eq. (2) is  $300 \times 300$ , while that for Eq. (7) is 100. Further, we chose  $G_5 \sigma = 3 \times 10^{-2} n$  and  $R_s = G_5 \sigma / 6 = 5 \times 10^{-3} n$ , where  $n = 1, 2, 3$ . This choice of  $\sigma$  guarantees  $|\Omega| \ll 1$  at the numerical boundaries  $R = R_{\max}$  and  $\xi = \xi_{\max}$ .

We have checked our numerical codes for the initial data and for marginal surfaces by comparing the

$l = \infty$  numerical solution of the energy density (12) with the  $l = \infty$  analytic solution

$$\Omega(R, \xi) = \frac{2G_5\sigma}{3r}. \quad (13)$$

The solution (13) is obtained by solving the Hamiltonian constraint (2) with the boundary condition (3) with a singular line source  $\rho(R)(\Omega + 1)^2 = \sigma\delta(R)/(2\pi R)$  under the situation  $l \rightarrow \infty$ .

We found both brane- and bulk-MS in the cases of all  $n$  (see Fig. 1). We also confirmed numerically that the inequality (6) holds in the cases of  $n = 1$  and  $n = 2$ , while it does not in the case of  $n = 3$ . Then we can say that *the bulk-MS can form even for so highly elongated matter distribution that the inequality  $C > 4\pi GM$  is satisfied*.

In Fig. 1 the bulk-MS does not agree with the brane-MS at the brane in our example. The same result is also obtained in Ref. [11]. To consider this behavior of the bulk- and brane-MS, it is instructive to compare with the analytic solution (13). In this solution, also, the bulk-MS ( $r = G_5\sigma/3$ ) does not agree with the brane-MS ( $R = 2G_5\sigma/3$ ) at the brane ( $\xi = 0$ ). On the other hand, in the case of the static black string solution  $\text{Sch} \times \mathbf{R}$  [10], the intersection of bulk-MS with the brane agrees with the brane-MS.

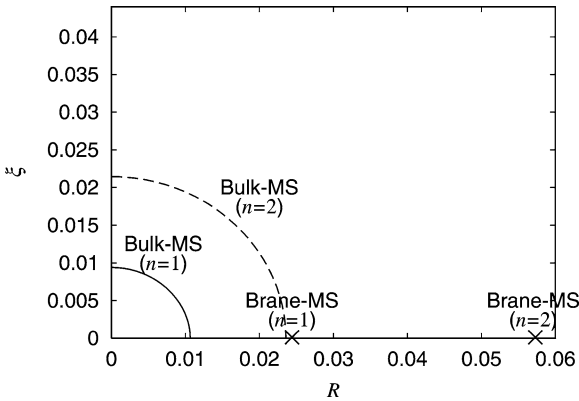


Fig. 1. We set  $G_5\sigma = 3 \times 10^{-2}n$  and  $l = 1$ . Then bulk-MS of  $n = 1, 2, 3$  and the location of brane-MS of  $n = 1, 2$  are depicted. Brane-MS of  $n = 3$  is located outside this figure ( $R \sim 0.132$ ). The matter is located within  $0 \leq R < G_5\sigma/6 = 5 \times 10^{-3}n$  at  $\xi = 0$ . From this figure, we can see that  $n = 1$  case almost agree with the  $l = \infty$  analytic solution; bulk- and brane-MS of  $l = \infty$  are given by  $r = G_5\sigma/3$  and  $R = 2G_5\sigma/3$ , respectively.

This difference suggest the existence of gravitational waves on the initial data considered here.

The existence of the cylindrically symmetric marginal surface shows that the inequality  $C \lesssim 4\pi GM$  is not a necessary condition for the formation of black holes with horizons in the RS scenario. This inequality will be just the sufficient condition for the black hole formations in the RS scenario. This suggests that massive spindle singularities in the RS brane is enclosed by event horizons if the existence of future null infinity and the global hyperbolicity of its causal past are guaranteed, while those in 4-dimensional general relativity do not. We say that the hoop conjecture, which seems to be natural in the original 4-dimensional Einstein gravity, does not hold in the RS brane world. We must note that this conclusion is not the extrapolation from the linear analyses but the result based on the general relativistic treatment without any weak field assumption. Using this treatment, we have obtained the qualitatively same result as the speculation based on the linear analysis.

From physical viewpoint, the existence of naked singularities must be interpreted as the appearance of some new fundamental physics. In this sense, for the gravitational collapse of matter elongated sufficiently, the hoop conjecture in the original 4-dimensional Einstein gravity can be used to judge the appearance of the new fundamental physics. As discussed here, the formation of the highly elongated marginal surfaces which violates the 4-dimensional hoop conjecture means the “signal” of the RS brane world. Probably, this “signal” may be not so significant in the astrophysical phenomena like collapse of the cylindrically distributed cosmic dust, because the stringlike dust concentration with the thickness less than  $l$  is necessary to form highly elongated marginal surfaces. However, we may expect that such phenomena happened on the cosmic strings formation in some era of very early universe (Davis treated early cosmic strings in the RS scenario within the linear analysis [16]). If so, the scenario of universe evolution may be modified by considering such phenomena, and in the future observational data of the Universe, we will find the “signal” which shows the possibility of the RS brane world.

In this Letter we presented only one important result on the hoop conjecture in the RS brane scenario. We may expect other appropriate criteria for the

black hole formation in the RS scenario. For further understanding and application of this result, more detailed investigations should be done. We should clarify the dependence on  $G_5\sigma/l$  of the formation of marginal surface to apply the result for cosmology, for example. We also note that our result does not mean necessarily that no naked singularity forms in the RS scenario in its original sense. The pancake-type gravitational collapse might lead to serious naked singularities in the RS model, although it is not so serious in the framework of 4-dimensional general relativity. The “signal” of the appearance of the new fundamental physics would be the pancake-type singularities on the brane rather than the stringlike matter distributions discussed here. These issues are future problems and will be discussed elsewhere.

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## References

- [1] K.S. Thorne, in: J. Klauder (Ed.), *Magic Without Magic*: John Archibald Wheeler, Freeman, San Francisco, 1972, p. 231.
- [2] R. Penrose, *Riv. Nuovo Cimento* 1 (1969) 252.
- [3] T. Nakamura, K. Maeda, S. Miyama, M. Sasaki, in: R. Ruffini (Ed.), *Proceedings of the 2nd Marcel Grossmann Meeting on General Relativity*, North-Holland, Amsterdam, 1982, p. 675.
- [4] S.L. Shapiro, S.A. Teukolsky, *Phys. Rev. Lett.* 66 (1991) 994.
- [5] L. Randall, R. Sundrum, *Phys. Rev. Lett.* 83 (1999) 4690.
- [6] M. Sasaki, T. Shiromizu, K. Maeda, *Phys. Rev. D* 62 (2000) 024008;  
T. Shiromizu, K. Maeda, M. Sasaki, *Phys. Rev. D* 62 (2000) 024012.
- [7] J. Garriga, T. Tanaka, *Phys. Rev.* 84 (2000) 2778.
- [8] G.C. Long, H. Chang, J. Price, *Nucl. Phys. B* 529 (1999) 23;  
C.D. Hoyle, U. Schmidt, B.R. Heckel, E.G. Adelberger, J.H. Gundlach, D.J. Kapner, H.E. Swanson, *Phys. Rev. Lett.* 86 (2001) 1418.
- [9] S.B. Giddings, E. Kats, L. Randall, *J. High Energy Phys.* 03 (2000) 023.
- [10] R. Gregory, R. Laflamme, *Phys. Rev. Lett.* 70 (1993) 2837;  
R. Gregory, *Class. Quantum Grav.* 17 (2000) L125.
- [11] T. Shiromizu, M. Shibata, *Phys. Rev. D* 62 (2000) 7502.
- [12] W. Israel, *Nuovo Cimento B* 44 (1966) 1;  
W. Israel, *Nuovo Cimento B* 48 (1967) 463;  
W. Israel, *Phys. Rev.* 153 (1967) 1388.
- [13] M. Yamaguchi, N. Kitazawa, private communications.
- [14] R. Emparan, G.T. Horowitz, R.C. Myers, *J. High Energy Phys.* 01 (2000) 007.
- [15] T. Nakamura, S.L. Shapiro, S.A. Teukolsky, *Phys. Rev. D* 38 (1988) 2972;  
J. Wojtkiewicz, *Phys. Rev. D* 41 (1990) 1867;  
E. Flanagan, *Phys. Rev. D* 46 (1992) 1429;  
A.M. Abrahams, K.R. Heiderich, S.L. Shapiro, S.A. Teukolsky, *Phys. Rev. D* 46 (1992) 2452;  
E. Malec, *Phys. Rev.* 67 (1991) 949;  
J. Wojtkiewicz, *Class. Quantum Grav.* 9 (1992) 1255;  
T.P. Tod, *Class. Quantum Grav.* 9 (1992) 1581;  
T. Chiba, T. Nakamura, K. Nakao, M. Sasaki, *Class. Quantum Grav.* 11 (1994) 431;  
C. Barrabès, A. Gramain, E. Lesigne, P.S. Letelier, *Class. Quantum Grav.* 9 (1992) L105.
- [16] S.C. Davis, *Phys. Lett. B* 499 (2001) 179.